How white is the sky?

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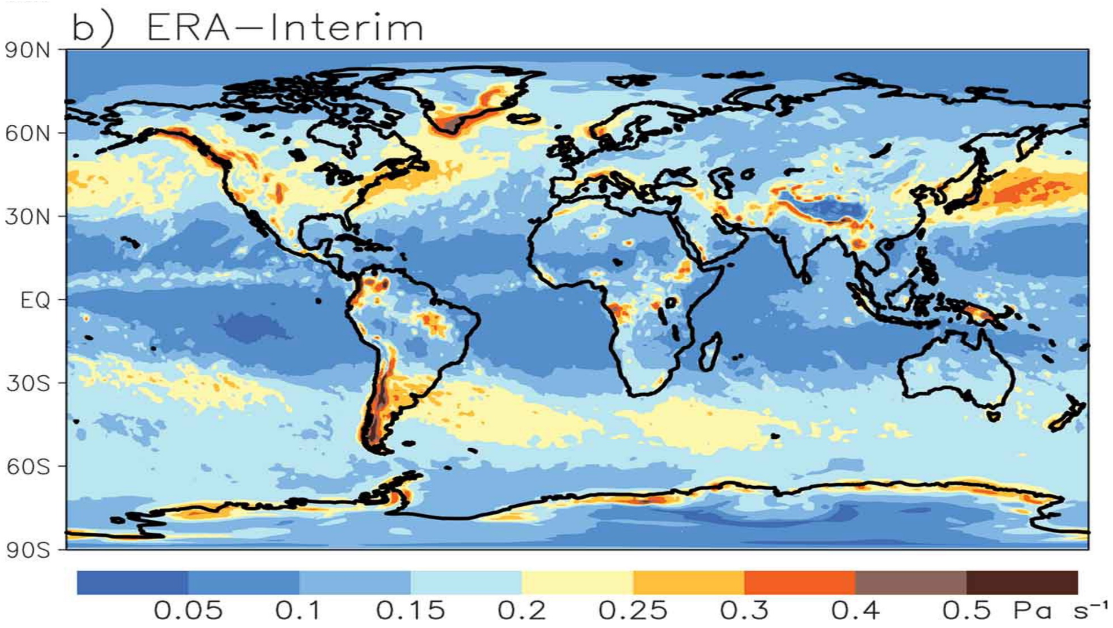
DER FORSCHUNG | DER LEHRE | DER BILDUNG
TRR181
ECMWF

AGU FALL MEETING
SCIENCE LEADS THE FUTURE

Vertical velocity in the hydrostatic atmosphere

$$\omega(\lambda,\varphi,\eta) = -\int_0^{\eta} \nabla_\eta \cdot \left(\mathbf{V}_H \frac{\partial p}{\partial \eta} \right) \mathrm{d}\eta + \mathbf{V}_H \cdot \nabla_\eta p$$

Computation of the vertical velocity on the ECMWF hybrid model level η



Stepanyuk et al., 2017

Root-mean-square (RMS) amplitude of omega at 600 hPa for the 12-month period in ERA-Interim reanalysis

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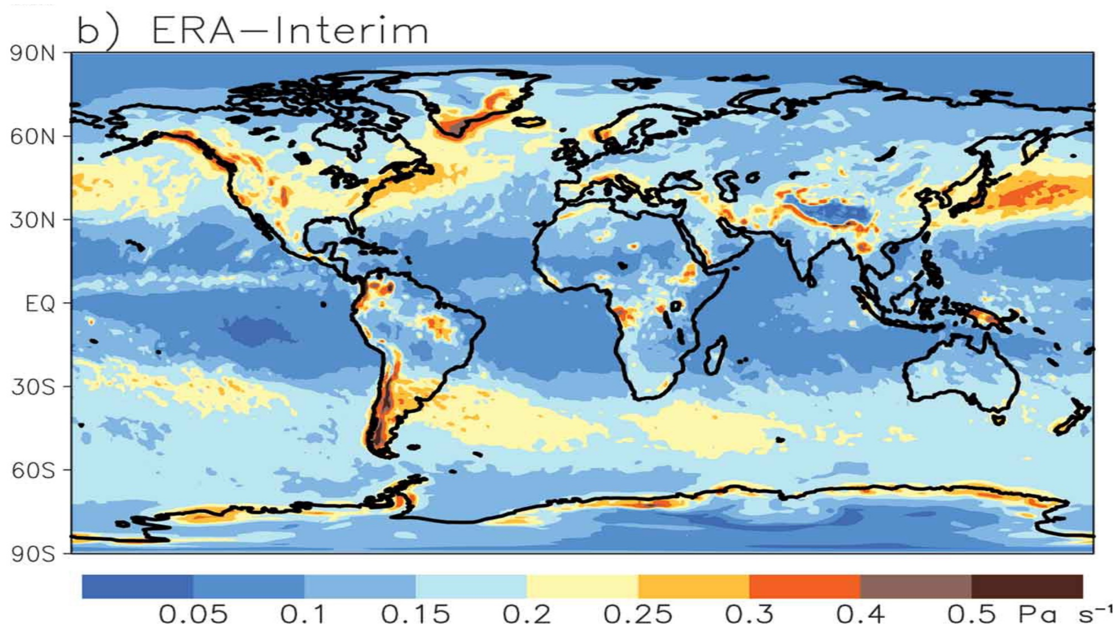


Stepanyuk et al., 2017

Root-mean-square (RMS) amplitude of omega at 600 hPa for the 12-month period in ERA-Interim reanalysis

Can we quantify vertical velocities associated with the Rossby and gravity waves?

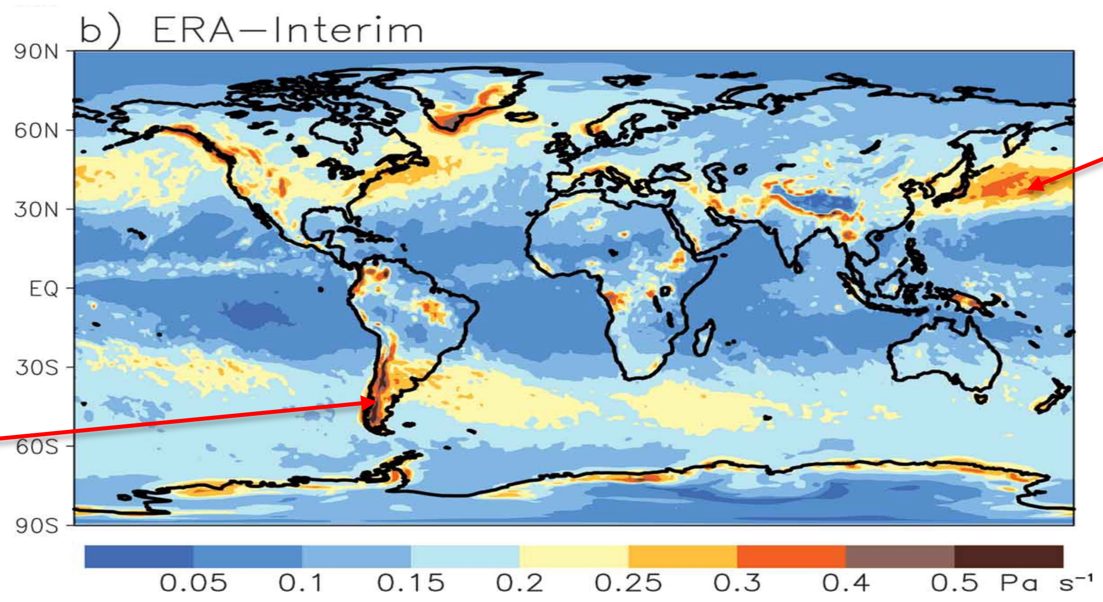
Rossby and gravity wave vertical motions

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_o^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\omega \approx \frac{f_o}{\sigma}\frac{\partial \mathbf{V_g}}{\partial p}\cdot\nabla\left(\frac{2}{f_o}\nabla^2\varphi + f\right)$$

Quasi-geostrophic (QG) omega equation

$$\frac{\mathrm{d}^2\tilde{w}}{\mathrm{d}^2 z} + \left[\frac{N^2}{(c-\bar{u})^2} + \frac{1}{c-\bar{u}}\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}^2 z} - \frac{1}{H}\frac{1}{c-\bar{u}}\frac{\mathrm{d}\bar{u}}{\mathrm{d}z} - \frac{1}{4H^2} - k^2\right]\tilde{w} = 0$$

Taylor-Goldstein equation (TGE)



QG omega equation

TGE

Stepanyuk et al., 2017

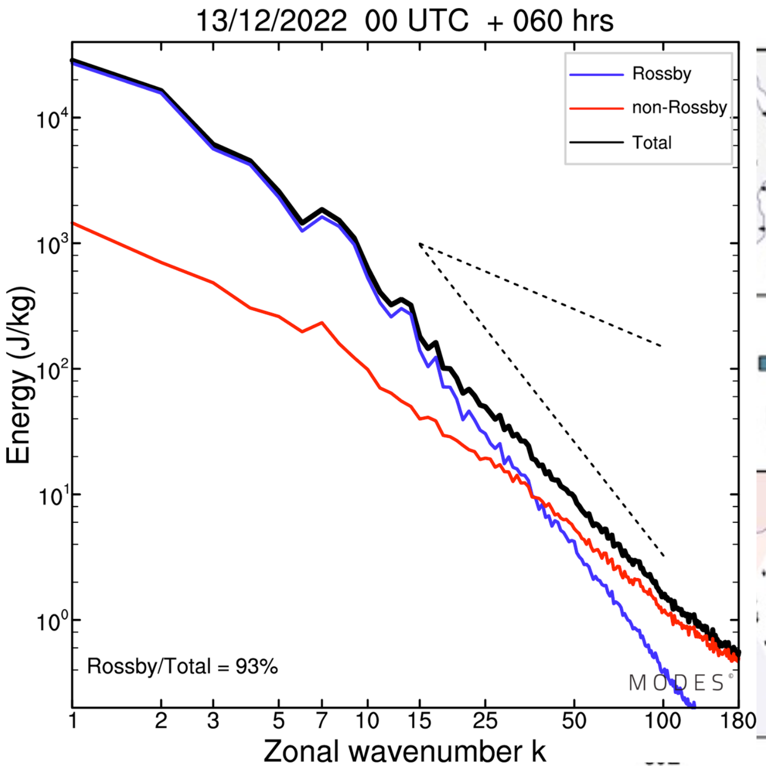
Tropics: QG theory breaks down, Kelvin and MRG waves play a significant role in dynamics.

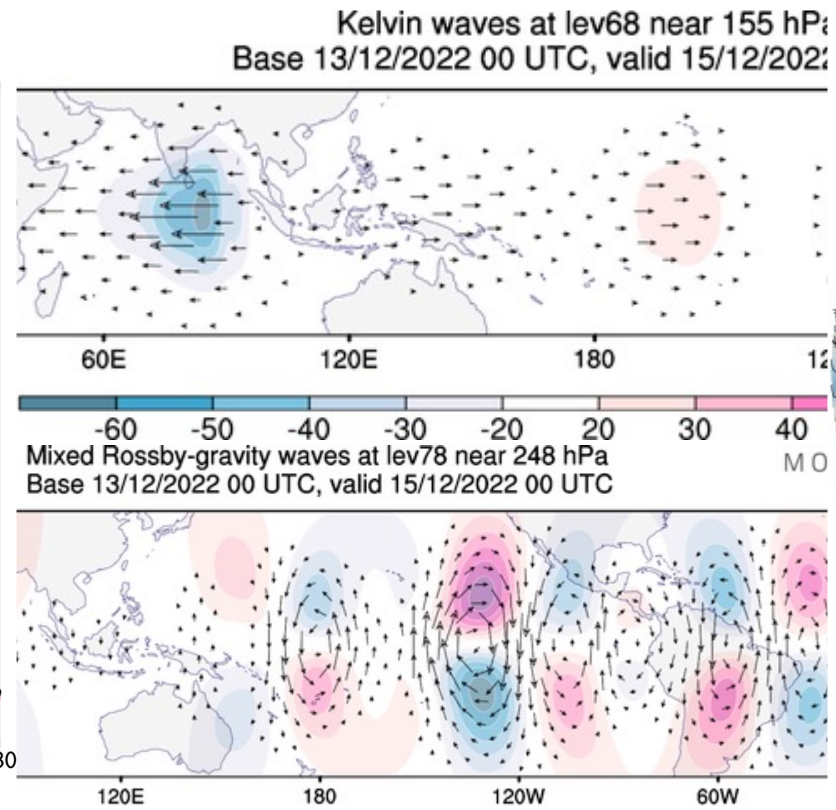
How can we systematically decompose vertical motions in the tropics?

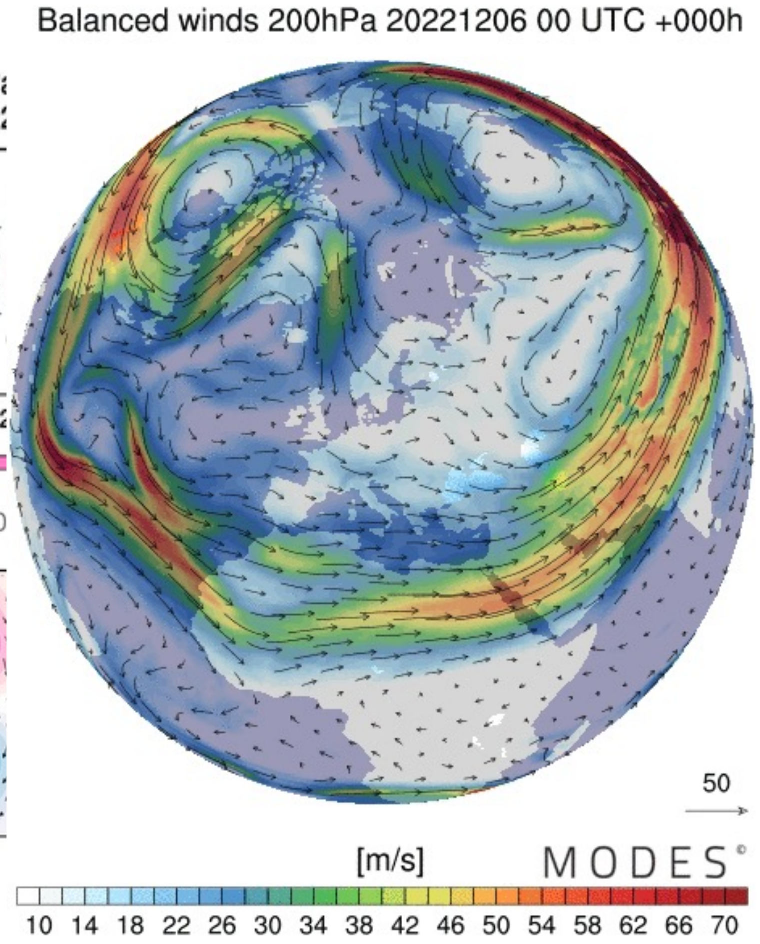
Unified framework for the vertical motions due to the Rossby and non-Rossby modes

Normal-mode function decomposition of linearized primitive equations

MODES - Real-time decomposition of ECMWF forecasts







http://modes.cen.uni-hamburg.de

Unified framework for the vertical motions due to the Rossby and non-Rossby modes

$$\tilde{h}(\lambda,\varphi) = \sum_{n=1}^{R}\sum_{k=-K}^{K} \chi_n^k Z_n^k(\varphi)\, e^{ik\lambda} e^{-i\nu_n^k t}$$

$\chi^k{}_n$ - complex Hough expansion coefficient

k - zonal wavenumber

n - meridional mode index

$$\tilde{\nabla}\cdot\tilde{\mathbf{V}} = \sum_{n=1}^{R}\sum_{k=-K}^{K} i\nu_n^k \chi_n^k Z_n^k(\varphi)\, e^{ik\lambda}$$

Horizontal wind divergence

(~ means non-dimensional quantities)

Modal component (k,n,m) pressure vertical velocity at a point (λ,φ,p)

$$\omega_n^k(m)(\lambda,\phi,p) = -\int_0^p 2\Omega i\nu_n^k(m)\chi_n^k(m) Z_n^k(\varphi;m)\, e^{ik\lambda}\, G_m(p')\,\mathrm{d}p'$$

m – vertical mode

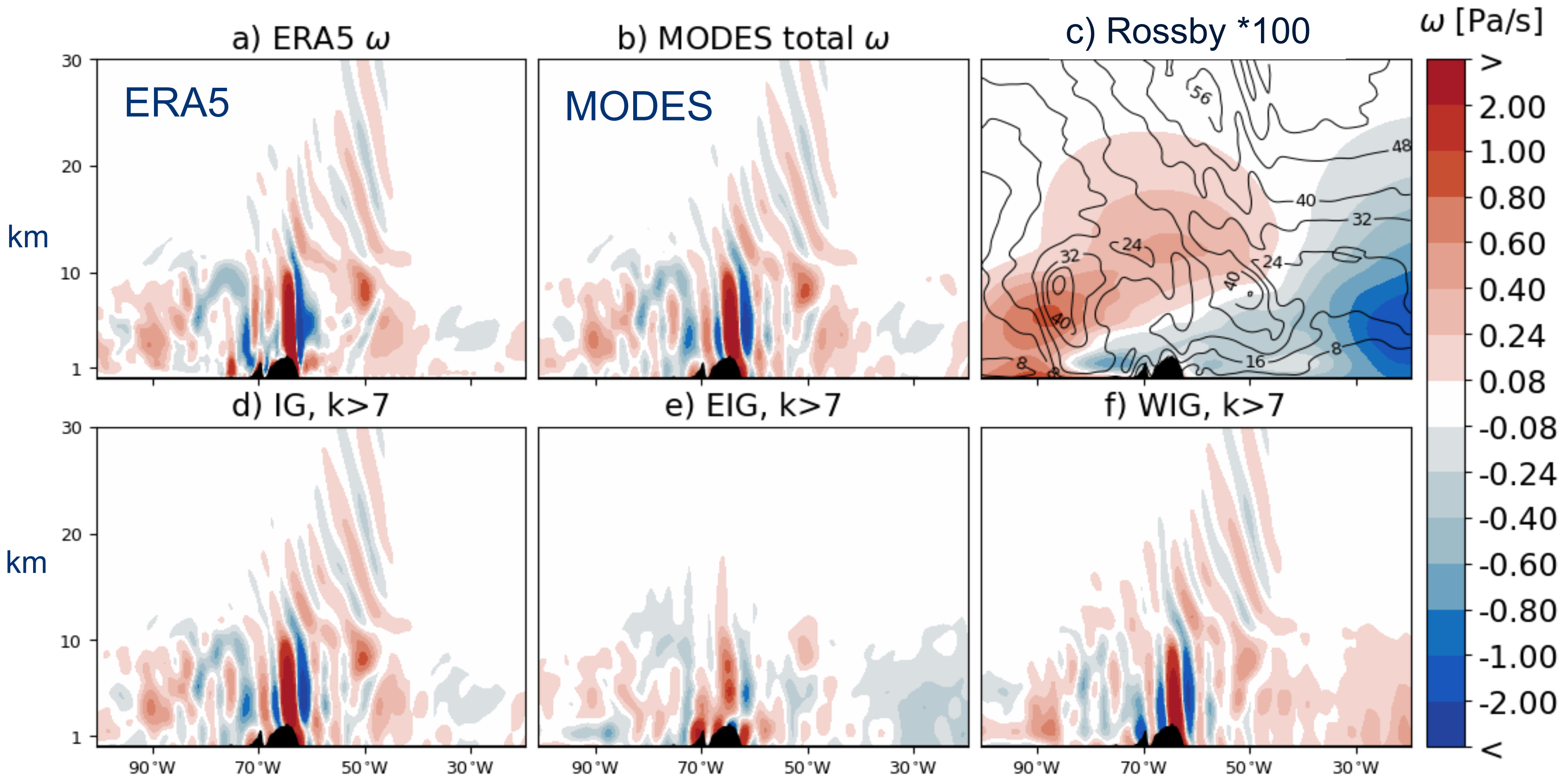
Frequency

Hough function for the geopotential height

Vertical structure functions

Regime decomposition of omega: an example

Vertical cross-section of omega along 70°S on 11 Aug 2018



Omega derived using the normal-mode approach agrees with the ERA5 omega

Unified framework for the vertical motions due to the Rossby and non-Rossby modes

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$$\omega(\lambda,\varphi,p) = \sum_{k=-K}^{K} \hat{\omega}_k(\varphi,p) e^{ik\lambda}$$

$$\hat{\omega}_k(\varphi,p) = -2\Omega i \sum_{m=1}^{M} \sum_{n=1}^{R} \nu_n^k(m)\chi_n^k(m) Z_n^k(\varphi;m) \int_0^p G_m(p')\mathrm{d}p'$$

Regime-dependent vertical kinetic energy spectra

Kinetic energy spectrum of vertical motions

$$K_\omega(\varphi,p) = K_n^k(\varphi,p,m) \propto \left(\nu_n^k(m)\right)^2 \left|\lambda_n^k(m)\right|^2 \propto \nu^2 E_H(\varphi,p)$$

Vertical kinetic energy

Frequency

Energy of horizontal motions

Dispersion relationships

$$\nu^2 + \frac{k\nu}{n(n+1)} - \frac{n(n+1)}{\varepsilon} = 0$$ Inertia-gravity modes

$$\nu = \frac{2k\Omega}{n(n+1)}$$ Rossby modes (R-H wave)

Rossby modes $\nu^2 \propto \kappa^{-2}$

Inertia-gravity modes $\nu^2 \propto \kappa^2$

$$E_H \propto \kappa^{-3} \Rightarrow K_\omega \propto \kappa^{-5}$$

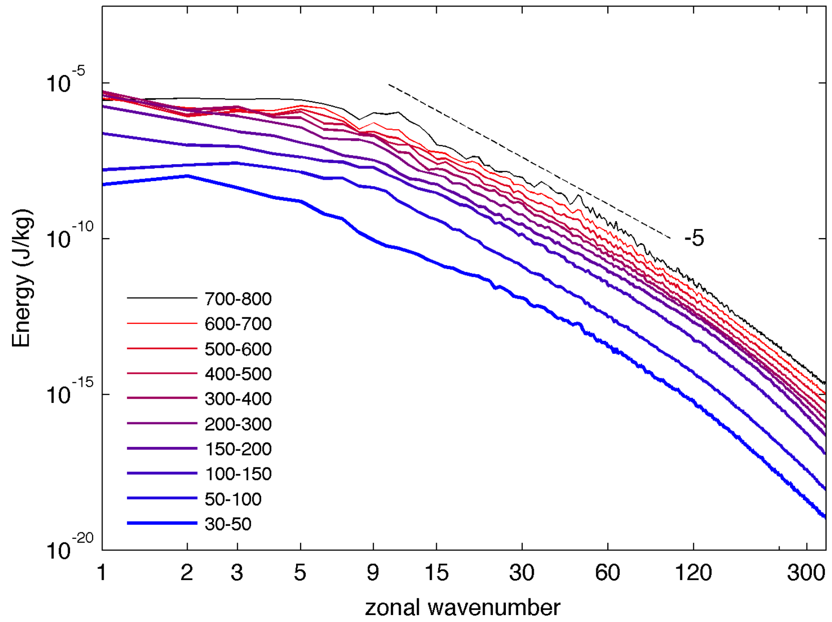
$$E_H \propto \kappa^{-5/3} \Rightarrow K_\omega \propto \kappa^{1/3}$$

$$K_\omega \Rightarrow K_\omega^R + K_\omega^{nR}$$

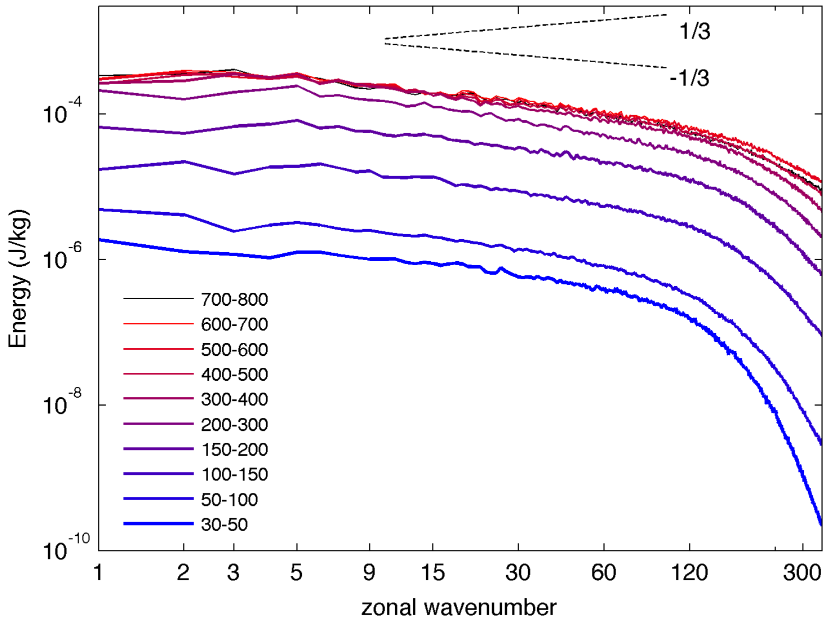
Application to ERA5: tropics

R – Rossby modes
nR – non-Rossby modes (inertia-gravity, Kelvin and mixed Rossby-gravity waves)

K_ω^R



K_ω^{nR}

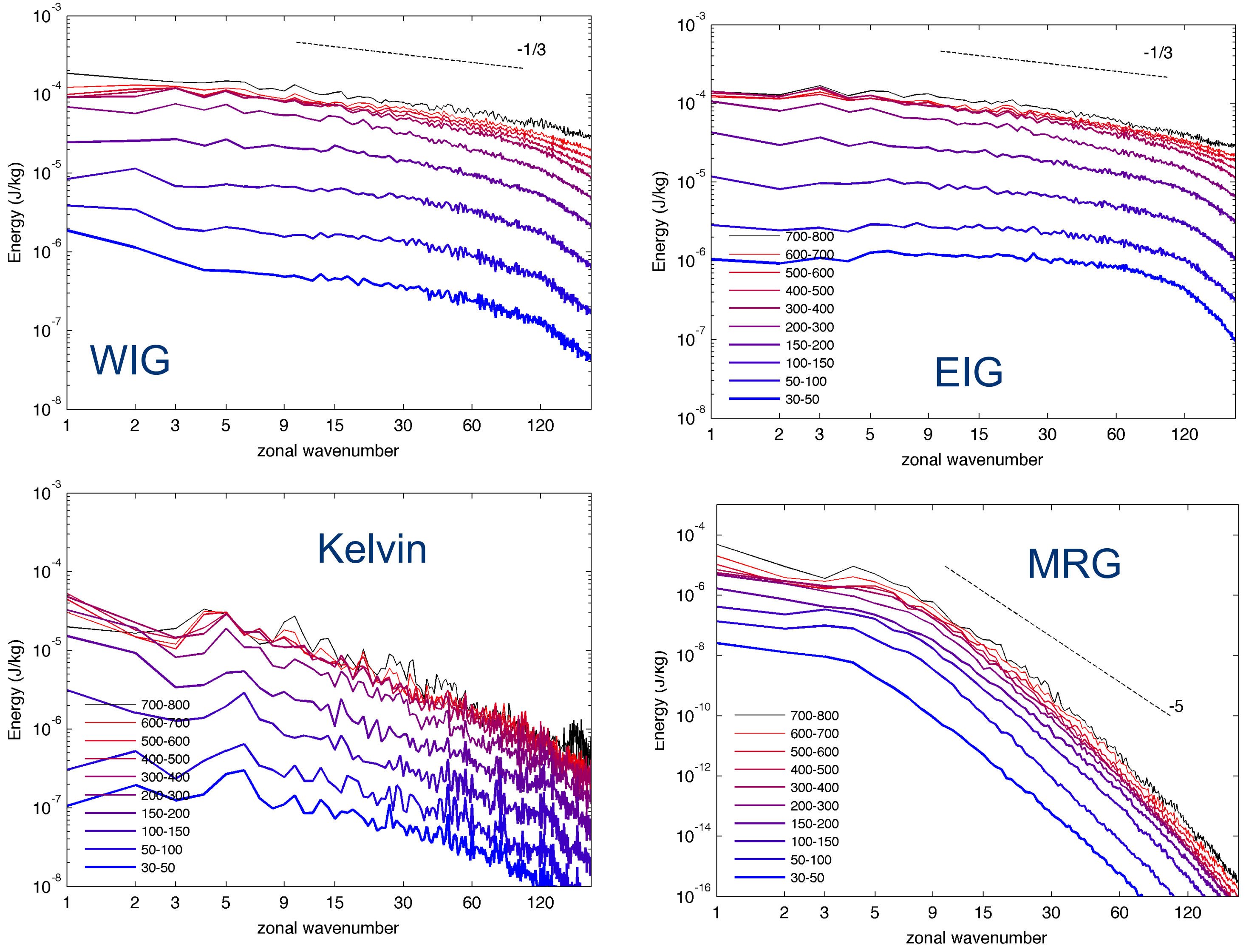


Average over latitude belt 10°S – 20°N for August 2018, data once per day

y-axis for the R modes has twice as many order of magnitude as the y-axis for the nR modes

Vertical kinetic energy spectra of the non-Rossby modes in the tropics

K_{ω}^{nR}



Average 10°S – 20°N
ERA5 August 2018
Data once per day
Easterly QBO phase

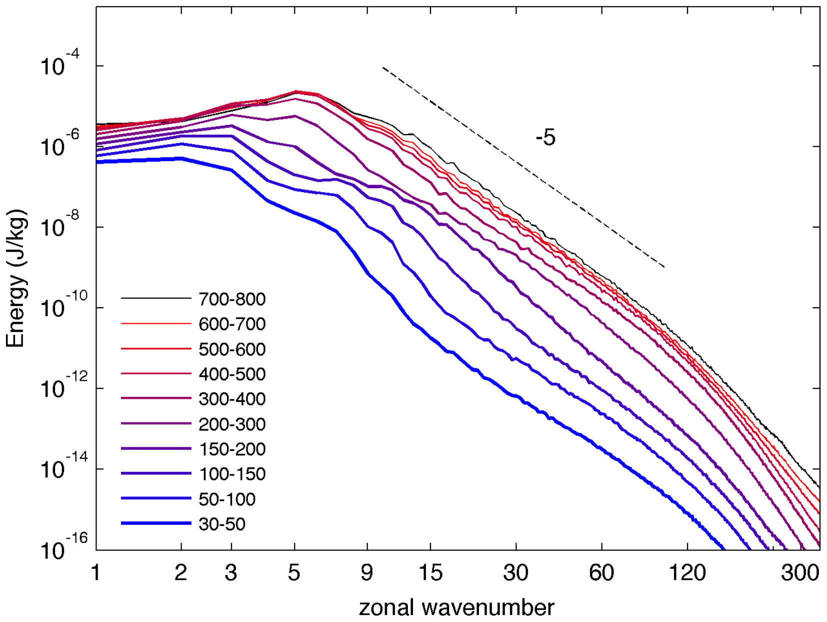
K_{ω}^{nR} associated with the Kelvin waves exceeds that of the IG modes at k=1 within the TTL

Y-axis for the MRG modes goes to e-16, other y-axes to e-8

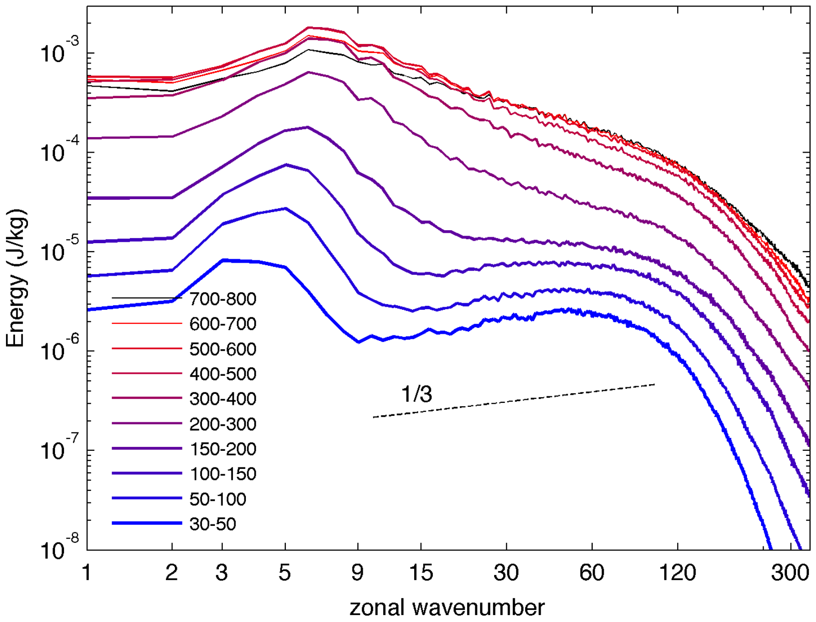
$K_\omega \Rightarrow K_\omega^R + K_\omega^{nR}$ Application to ERA5: extratropics

R – Rossby modes
nR – non-Rossby modes (inertia-gravity, Kelvin and mixed Rossby-gravity waves)

K_ω^R



K_ω^{nR}



Average over latitude belt 30°S – 60°S for August 2018

y-axis for the R modes has twice as many order of magnitude as the y-axis for the nR modes

Summary

- A unified approach to the regime decomposition of the vertical velocity in the hydrostatic atmosphere has been derived. The motivation is the decomposition of the vertical velocity and momentum fluxes in the tropics.

- Kinetic energy spectra of vertical motions are proportional to the square of the frequency of the eigenmodes of linearized primitive equations.

- Based on the -3 and -5/3 power laws of the energy spectra of horizontal motions, the vertical kinetic energy spectra follow the -5 and 1/3 power laws for the Rossby and non-Rossby parts, respectively.

- Application to ERA5 highlights a lack of gravity wave variance at subsynoptic scales.

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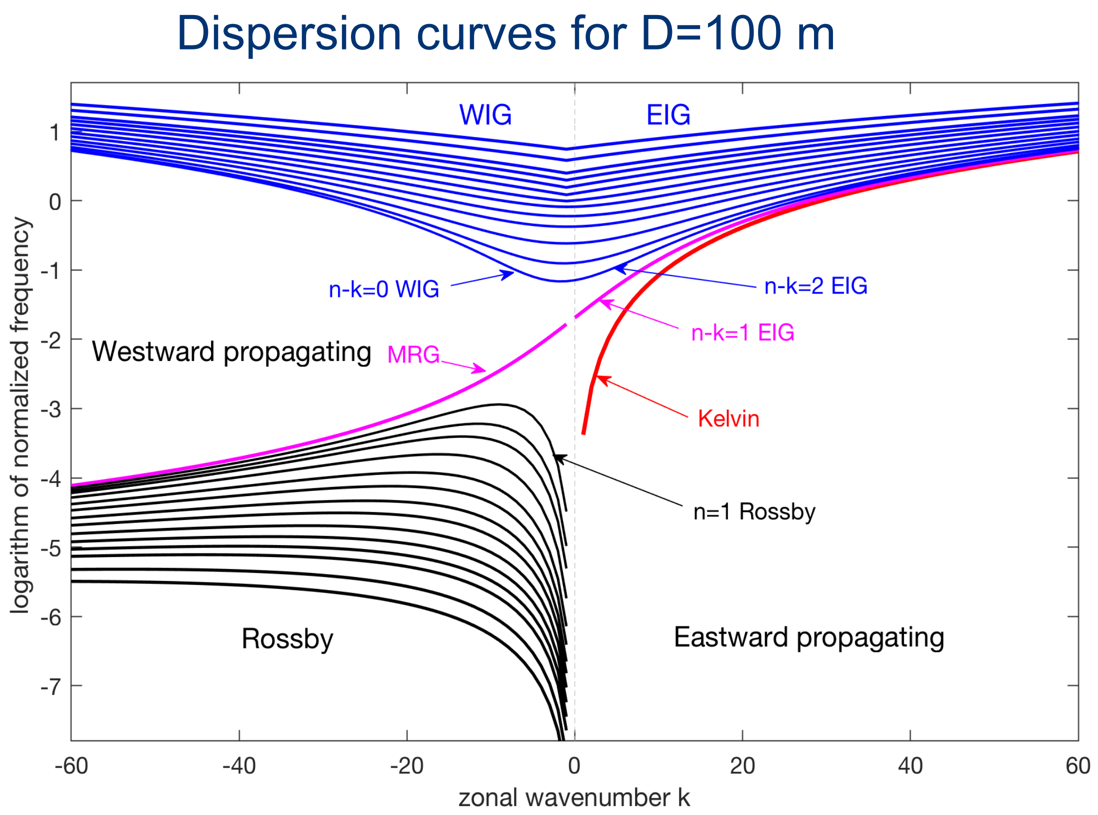
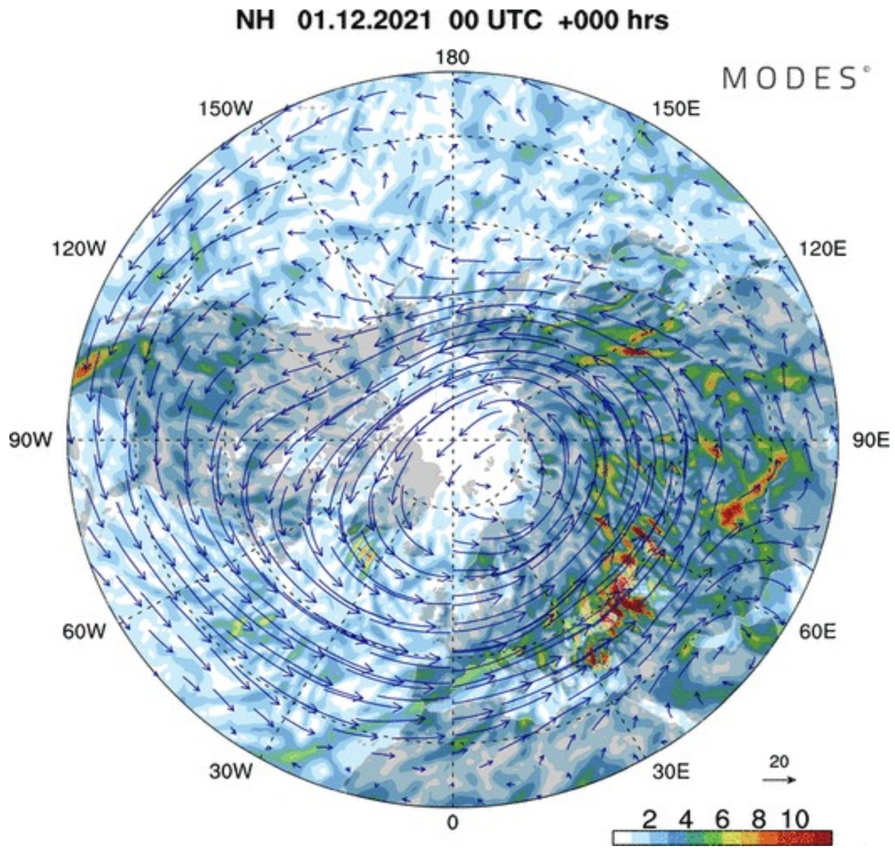
Additional slides

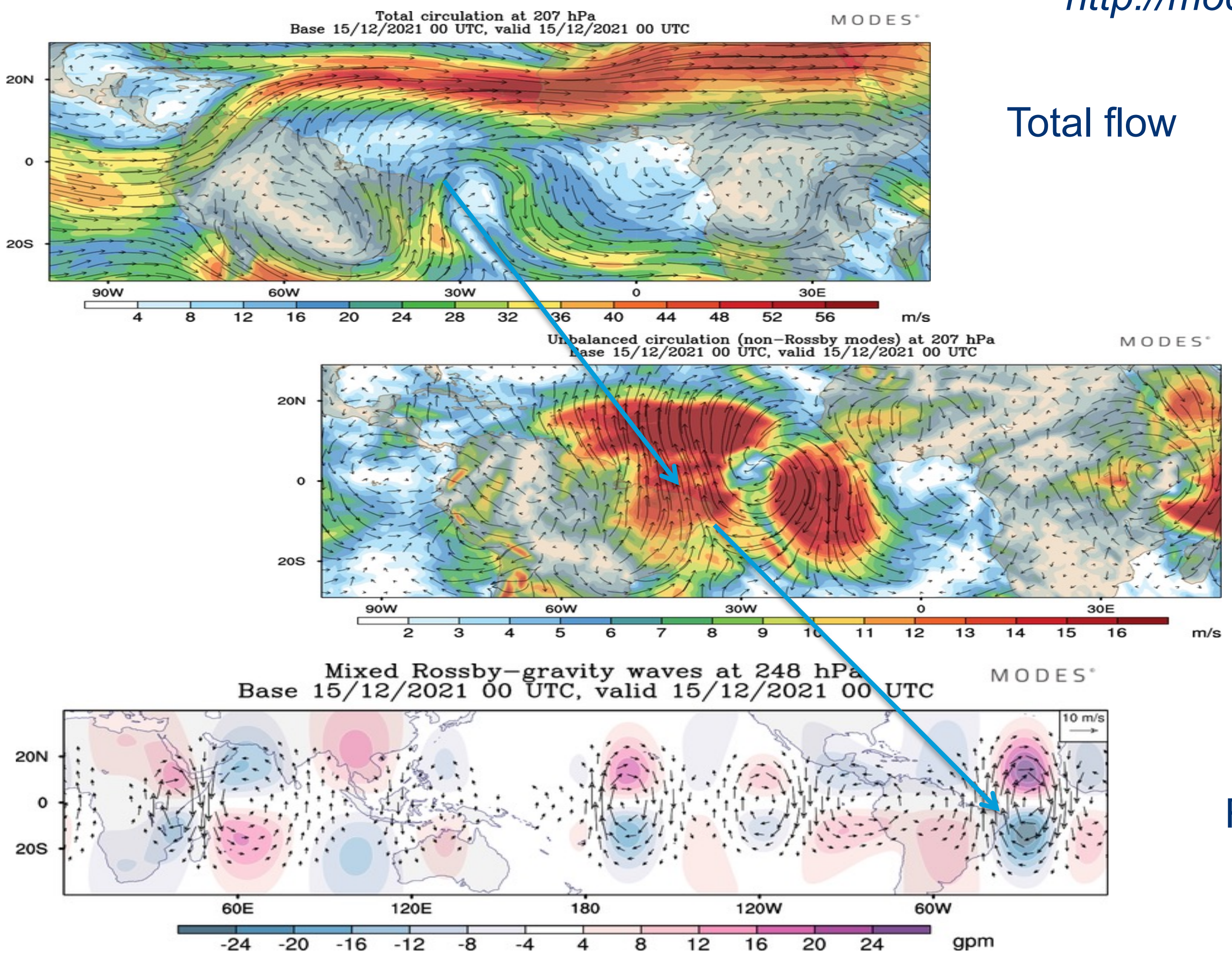
MODES


Dispersion curves for D=100 m
WIG
EIG
n-k=0 WIG
n-k=2 EIG
n-k=1 EIG
Westward propagating
MRG
Kelvin
n=1 Rossby
Rossby
Eastward propagating
logarithm of normalized frequency
zonal wavenumber k
-60
-40
-20
0
20
40
60
1
0
-1
-2
-3
-4
-5
-6
-7


NH 01.12.2021 00 UTC +000 hrs
MODES
180
150W
150E
120W
120E
90W
90E
60W
60E
30W
30E
0
20
2 4 6 8 10

Tropical circulation decomposed: an example

http://modes.cen.uni-hamburg.de



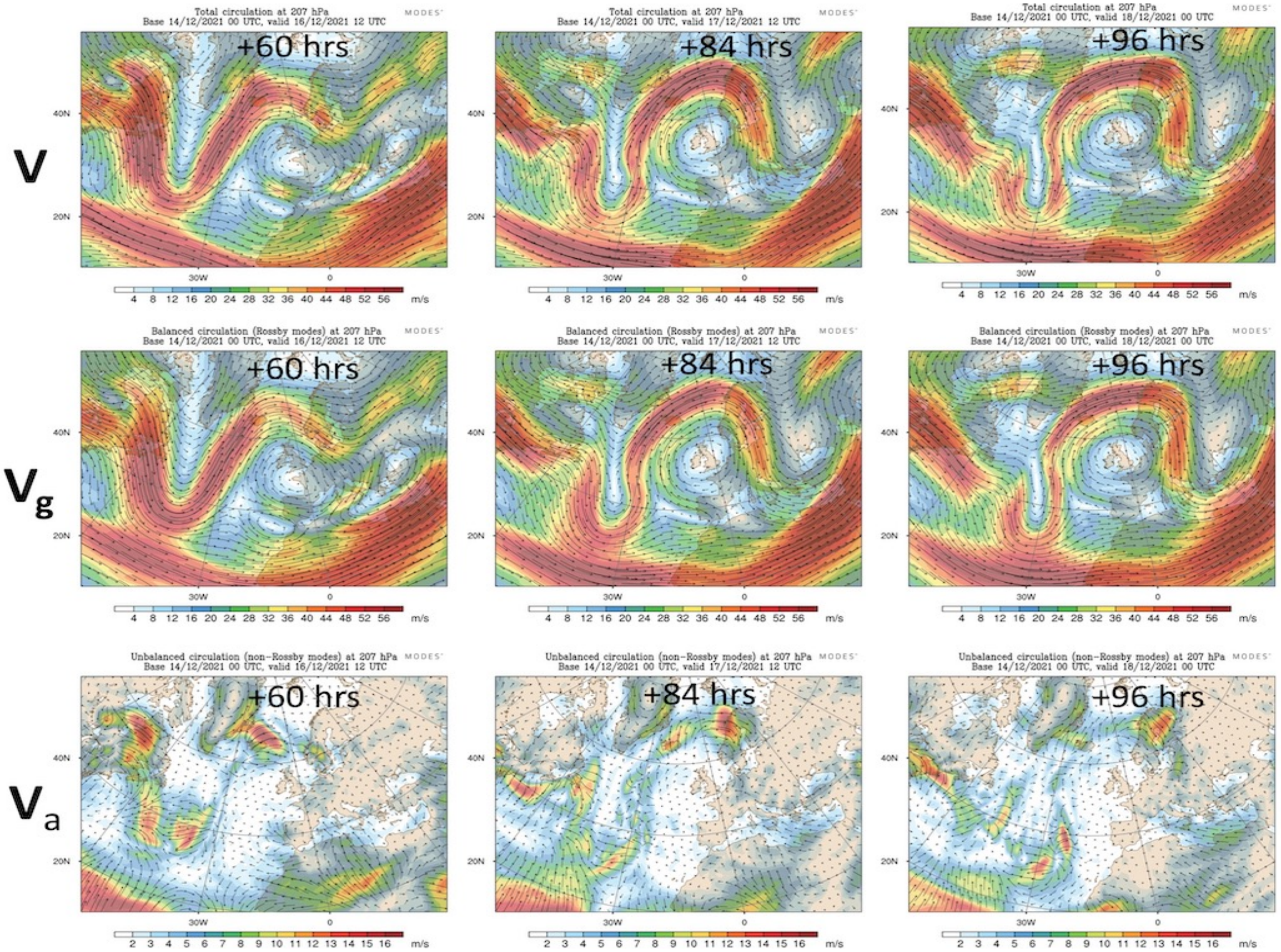
Total flow

Unbalanced flow

Rossby-gravity wave signal

Non-Rossby modes & ageostropic circulation

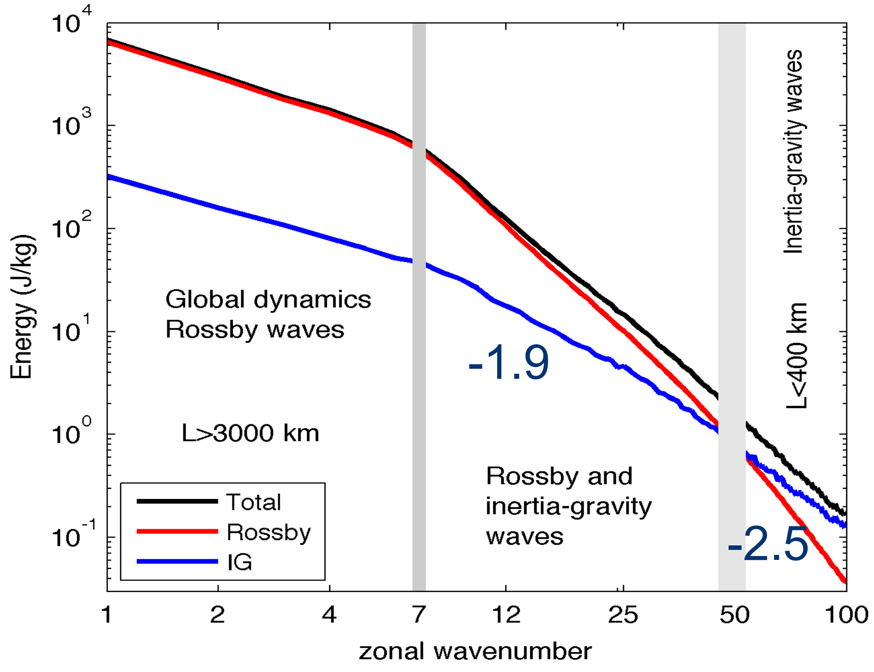
$$\mathbf{V}_a = \frac{1}{f}\mathbf{k} \times \frac{d\mathbf{V}}{dt}$$



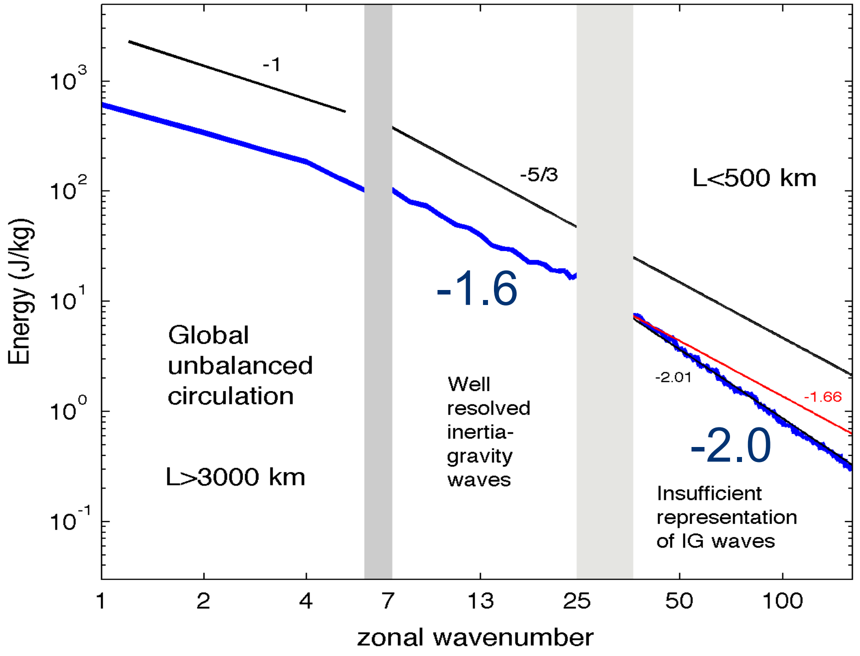
Global energy spectra (K+APE) from MODES

Advancements in simulated spatial variability

ERA Interim, 35-year average



Inertio-gravity mode spectrum
(oper ECMWF an, 2015)



Zagar et al., J. Atmos. Sci., 2017